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These works were written by 192 different authors, 125 of whom produced only one book or paper each. The largest number written by one author is 14; this author was Laplace. Out of the 404 works I have seen and actually inspected 309.

Like all bibliographical researches of this kind my list cannot be regarded as complete. With better library facilities the number of titles in the Italian, Dutch and Scandinavian languages would be much increased; and one who can consult the Russian and Hungarian literature might undoubtedly find a few titles to add although the articles themselves would probably not be of great value. I hope at some future time to be able to publish the list accompanied with historical and critical notes giving brief accounts of the contents of each book or memoir with perhaps an estimate of its value.

PEDAL CURVES.

BY PROF. W. W. JOHNSON, ANNAPOLIS, MARYLAND.

WHEN the rolling curve is equal to the fixed curve, the curves touching at corresponding points, as in the Figure on page 71, No. 3, ANALYST, the locus of any point connected with the rolling curve is the same as the Pedal of the given curve, doubled in linear dimensions. For consider the point in the plane of the fixed curve corresponding to the generating point, as D in the figure; it is plain that D and D'' are symmetrical points with reference to the common tangent at P, so that DD'' is perpendicular to the tangent and bisected by it, and the locus of D'' is similar to that of the foot of the perpendicular upon the tangent, 2 being the ratio, and D the centre of similitude.

It may be remarked that since the tangent to the roulette, or locus of D'', and to the pedal are parallel, the well known and obvious property of roulettes that PD'' is a normal affords a simple proof of the property of the pedal that the inclination of its tangent and radius vector is the same as the corresponding angle at the point of contact P in the original curve.

When the equation of the tangent to the given curve, in terms of its direction ratio m, is known, the equation of the pedal is very easily found; for, (h, k) being the pedal origin, the equation of the perpendicular upon the tangent is

$$y - k = -\frac{1}{m}\left(x - h\right),$$

hence if we eliminate m between this equation and that of the tangent, by substituting in the latter $m = (h - x) \div (y - k)$, we shall at once have the quation of the pedal.

Thus for the parabola $y^2 = 4ax$, the tangent being $y = mx + (a \div m)$ the pedal is

$$y + x\frac{x-h}{y-k} + a\frac{y-k}{x-h} = 0.$$

The pedal origin satisfies the equation of the pedal even when it is not a point of the geometrical locus. The following explanation of this circumstance is taken from Briot and Bouquet's Ge'ome'trie Analytique. When a locus is determined by the intersection of two variable lines, A and B, whose equations are

$$f_1(x, y, a) = 0 \text{ and } f_2(x, y, a) = 0,$$

containing one arbitrary parameter a (or n parameters connected by $n - 1$ relations) the locus

$$f(x, y) = 0$$

is found by eliminating a. Now if the line A always passes through the fixed point (x_1, y_1) this point will satisfy $f(x, y) = 0$; for substitute x_1 and y_1 for x and y in the equation of B, we may determine from the result certain real or imaginary values of a; let a_1 be one of these values, then since, by hypothesis (x_1, y_1) satisfies the equation of A, no matter what the value of a, the values x_1 y_1 and a_1 satisfy the equations $f_1 = 0$ and $f_2 = 0$ and hence also $f = 0$ which is derived from them.

In the case of the pedal the fixed point is the pedal origin and since at least two tangents real or imaginary can be drawn through it, it is always a node. When the pedal origin is a focus however the equation breaks up into two factors, one of which denotes an infinitesimal circle at the pedal origin, hence this factor may be removed and the other represents only the geometrical locus which is a circle.

The following important proposition connects pedals with inverse curves:- If we take the polar reciprocal of a given curve with respect to a circle, the pedal to the given curve, with respect to the centre of the auxilliary circle, is the inverse of the reciprocal curve with respect to the same point. This is evident geometrically as follows; let O denote the pedal origin and OP' the perpendicular upon the tangent to the given curve, then the pole of the tangent with respect to a circle whose radius is k is a point P_1 so taken on the perpendicular OP' that $OP_1 \cdot OP' = k^2$. The polar reciprocal is the locus of P_1 and the pedal is the locus of P', but by the definition of inversion, these loci are mutually inverse curves.

The same thing may be proved analytically as follows, taking the origin of coordinates at the pedal origin, the polar of (x_1, y_1) with respect to the circle $x^2+y^2 = k^2$ is

$$xx_1 + yy_1 = k^2,$$

hence if the tangent to the given curve be put in this form, we can express x_1 and y_1 in terms of an arbitrary parameter and, by eliminating this parameter, find the equation of the locus of (x_1, y_1), which is the reciprocal curve. Now if (x', y') describes the inverse of this locus we have (as at p. 43, ANALYST, Vol. IV.)

$$x_1 = \frac{k^2x'}{x'^2+y'^2} \quad \text{and} \quad y_1 = \frac{k^2y'}{x'^2+y'^2},$$

whence the tangent line may be written in the form

$$xx'+yy' = x'^2+y'^2,$$

but this is the equation of a line passing through (x', y') and perpendicular to the line joining this point to the origin; that is, (x', y') is the foot of the perpendicular upon the tangent and describes the pedal to the given curve.

The reciprocal of a conic being a conic, it follows that the pedals to a conic are the inverse curves of another conic, that is they are either circles or (see p. 47, ANALYST, Vol. IV.) the nodal forms of the bicircular quartic and circular cubic. In particular, the central pedals and central inverse curves of the conic consist of the same class of curves, namely the symmetrical bicircular quartics with node at the centre of symmetry.

In the case of the rectangular hyperbola $x^2 - y^2 = a^2$, the reciprocal with respect to the circle $x^2+y^2 = a^2$ is identical with the given curve; for putting the equation of the tangent in the form

$$xx_2 - yy_2 = a^2, \tag{1}$$

where

$$x_2^2 - y_2^2 = a^2, \tag{2}$$

and comparing with the equation of the polar of (x_1, y_1)

$$xx_1+yy_1 = a^2, \tag{3}$$

we have

$$x_2 = x_1 \quad \text{and} \quad y_2 = -y_1,$$

and, substituting in (2), we have

$$x_1^2-y_1^2 = a^2$$

for the equation of the reciprocal. Hence the central pedal of the rectangular hyperbola is the same as its central inverse, namely the Lemniscate. The locus of the centre of a rectangular hyperbola rolling upon an equal hyperbola is therefore a lemniscate with axis double that of the hyperbola.

The theorem that the reciprocal of a conic with respect to a focus is a circle may be derived from the familiar property that the pedal of a conic with respect to a focus is a circle (or straight line in case of the parabola); for the inverse of the pedal is the reciprocal, and the inverse of a circle is known to be a circle.

The pedal to the circle is the limacon; for, taking the pedal origin as pole, and the centre of the circle (radius a) at the distance b on the initial line, the equation of the pedal is at once seen to be

$$r = a + b \cos \theta,$$

which is the polar equation of the limacon. Accordingly, the limacon is the inverse of the reciprocal of the circle, that is, it is the inverse of a conic with respect to a focus.

The reciprocal of a conic with respect to a point on the curve is a parabola; with respect to a point outside the curve (that is one from which real tangents can be drawn) the reciprocal is an hyperbola; and for a point within the curve, it is an ellipse.

Accordingly, the pedal with respect to a point on the curve, like the inverse of the parabola, is cuspidal; for a point without, it is crunodal, like the inverse of an hyperbola; and for a point within, it is acnodal, like the inverse of an ellipse. On the other hand, the pedal to the parabola, like the inverse with respect to a point on the curve, is a cubic, while the pedals of the other conics are quartics.

THE DIFFERENTIAL EQUATIONS OF PROBLEM 165.

BY PROF. A. HALL, NAVAL OBS., WASHINGTON, D. C.

SINCE the differential equations of this problem are those which furnish the theory of the pure elliptic motion of the planets they have been very completely discussed, and it seems worth while to notice a few of the properties that have been deduced. First, we may show without any integration that the planet moves in a plane. If we consider three successive positions of the planet whose coordinates are x, $x+dx$, and $x+2dx+d^2x$, with similar values for y and z; and form the determinant of the third order whose elements are these coordinates, by means of the given differential equations this determinant reduces to zero, which is the condition that the three points are in a plane.

One of the most elegant discussions of the equations is that given by Laplace, Mec. Cel., Book II. arts. 17 and 18. Laplace first notices that each of the equations may be written in the form

$$d.\left(r^3.\frac{d^2x}{dt^2}\right) + m.dx = 0,$$

and then shows that we have also the similar form

$$d.\left(r^3.\frac{d^2r}{dt^2}\right) + m.dr = 0. \tag{1}$$